TRINITY COLLEGE

Semester One Examination, 2023

Question/Answer booklet

MATHEMATICS APPLICATIONS UNIT 3

Section Two: Calculator-assumed

SOLUTIONS

WA student number: In figures | | | | | | | | | |

In words _____

_____

Your name _____

Time allowed for this section

Reading time before commencing work: ten minutes
Working time: one hundred minutes

Materials required/recommended for this section

To be provided by the supervisor

This Question/Answer booklet
Formula sheet (retained from Section One)

To be provided by the candidate

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: drawing instruments, templates, notes on two unfolded sheets of A4 paper, and up to three calculators, which can include scientific, graphic and Computer Algebra System (CAS) calculators, are permitted in this ATAR course examination

Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of examination
Section One: Calculator-free	5	5	50	51	35
Section Two: Calculator-assumed	10	10	100	96	65
				Total	100

Instructions to candidates

1. The rules for the conduct of Trinity College examinations are detailed in the *Instructions to Candidates* distributed to students prior to the examinations. Sitting this examination implies that you agree to abide by these rules.

2. Write your answers in this Question/Answer booklet preferably using a blue/black pen. Do not use erasable or gel pens.

3. You must be careful to confine your answers to the specific question asked and to follow any instructions that are specific to a particular question.

4. Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.

5. It is recommended that you do not use pencil, except in diagrams.

6. Supplementary pages for planning/continuing your answers to questions are provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.

7. The Formula sheet is not to be handed in with your Question/Answer booklet.

See next page

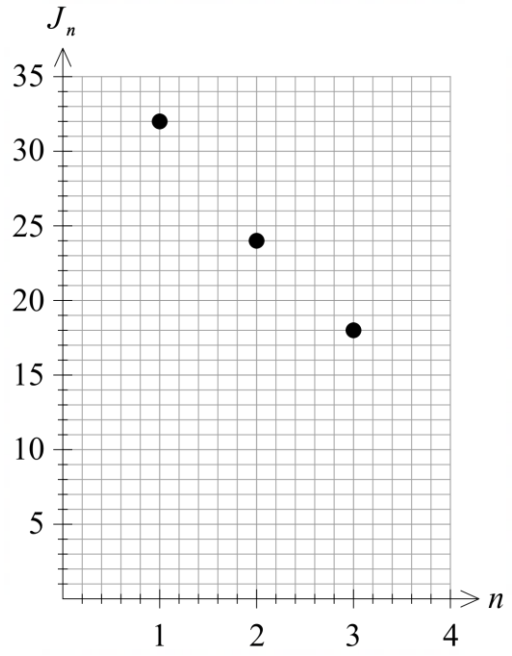
Section Two: Calculator-assumed **65% (96 Marks)**

This section has **ten** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time: 100 minutes.

Question 6 **(4 marks)**

The graph below shows the length of a frog's first three jumps, J_n, measured in centimetres.



(a) Determine the length of the frog's fifth jump. (2 marks)

Solution	Specific behaviours	Point
$32, 24, 18$ $J_{n+1} = 0.75J_n \quad J_1 = 32$ $J_5 = 10.125$ cm	✓ Determines sequence is geometric. ✓ Determines length of the fifth jump.	3.2.5 3.2.6

To conserve energy, the frog stops jumping once they jump less than 5 cm.

(b) Determine how many jumps the frog makes. (2 marks)

Solution	Specific behaviours	Point
$J_7 = 5.70$ cm $J_8 = 4.27$ cm Makes 8 jumps	✓ Continues sequence found in (a). ✓ Determines number of jumps.	3.2.5 3.2.6

See next page

Question 7 **(11 marks)**

Before the last federal election, a polling company asked 1500 people for who they intended to vote. The information is displayed below.

	Voting Intention				
	Labor	**Liberal**	**Independent**	**Other**	**Total**
Male	225	**205**	**250**	33	713
Female	**311**	177	277	22	**787**
Total	**536**	382	**527**	55	**1500**

(a) Complete the two-way table above. (2 marks)

Solution	Specific behaviours	Point
See table above	✓ At least four entries correct. ✓ All entries correct.	3.1.2

(b) Explain why 'Voting Intention' is not the explanatory variable. (1 mark)

Solution	Specific behaviours	Point
Voting Intention does not explain a person's gender.	✓ Explains that intention cannot influence a person's gender.	3.1.8

(c) An empty two-way percentage table is shown below.

	Voting Intention				
	Labor	**Liberal**	**Independent**	**Other**	**Total**
Male	**31.6%**	**28.8%**	**35.1%**	**4.6%**	**100%**
Female	**39.5%**	**22.5%**	**35.2%**	**2.8%**	**100%**
Total					

Complete the table by using either row **or** column percentages, as appropriate. (3 marks)

Solution	Specific behaviours	Point
See table above.	✓ Uses row percentages. ✓ Completes male row correctly. ✓ Completes female row correctly.	3.1.2

See next page

(d) For which candidate is there no association between the intention to vote for them and gender? (1 mark)

Solution	Specific behaviours	Point
Independent	✓ States independent.	3.1.3

(e) Describe **two** associations that can be observed from the two-way percentage table, giving reasons for your answer. (4 marks)

Solution	Specific behaviours	Point
Females are more likely to vote for Labor then Males (39.5% of females vs 31.6% of males.) Males are more likely to vote for Liberal then Females (22.5% of females vs 28.8% of males.) Males are more likely to vote for Other then Females (2.8% of females vs 4.6% of males.)	✓ States one association. ✓ Gives percentages from the table in (c) to support their answer. ✓ States second association. ✓ Gives percentages from the table in (c) to support their answer.	3.1.3 3.1.4

See next page

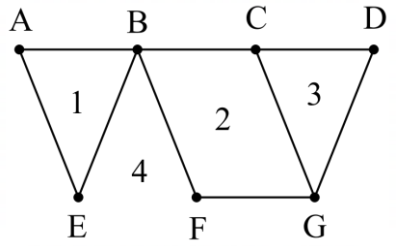
Question 8 **(6 marks)**

A connected planar graph has 4 faces and 9 edges.

(a) Determine the number of vertices for this network. (2 marks)

Solution	Specific behaviours	Point
$v + f - e = 2$ $v + 4 - 9 = 2$ $v = 7$	✓ Substitutes into Euler's formula. ✓ Determines number of vertices.	3.3.5

(b) Draw the connected planar network in the space below, labelling the vertices. (2 marks)

Solution	Specific behaviours	Point
	✓ Network has four faces and nine edges. ✓ Network has seven vertices.	3.3.4

n vertices and n edges are added to the network in part (b), where n is a positive integer.

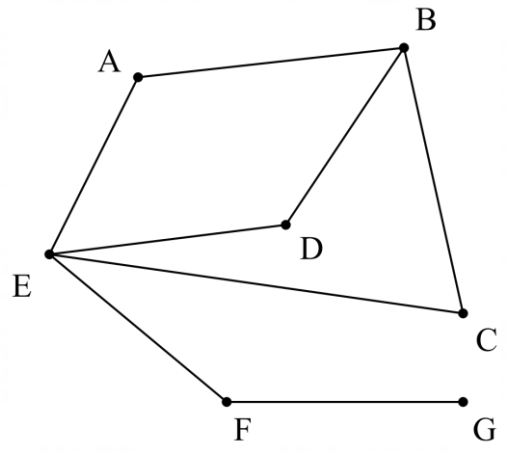
(c) If the network is still a planar connected network, does the number of faces change? Justify your answer. (2 marks)

Solution	Specific behaviours	Point
No, as the network is still planar then Euler's formula must hold. $7 + n + f - (9 + n) = 2$ $7 + n + f - 9 - n = 2$ The n's cancel out, giving $f = 4$	✓ States no. ✓ Recognises that Euler's formula still holds, and uses this to explain why the number of faces does not change.	3.3.4 3.3.5

See next page

Question 9 **(4 marks)**

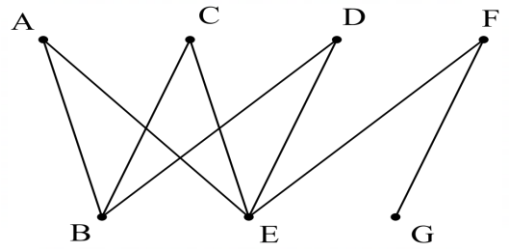
Consider the graph below.



(a) How many bridges does the graph have? (1 mark)

Solution	Specific behaviours	Point
2	✓ States the number of bridges.	3.3.6

(b) Show that the graph is clearly bipartite. (2 marks)

Solution	Specific behaviours	Point
	✓ Clearly splits the vertices into two groups. ✓ Places all eight edges on the graph.	3.3.1

The bipartite graph splits the vertices into two groups of students. The edges indicate that the two students are friends with each other.

A note is passed from student to student. Only friends can pass it to another friend.

(c) G wants to get a note to C. Explain why E has to be involved. (1 mark)

Solution	Specific behaviours	Point
As FG and EF are bridges, then any path from G to C must include E.	✓ States that FG and EF are bridges and hence E must be involved.	3.3.6

See next page

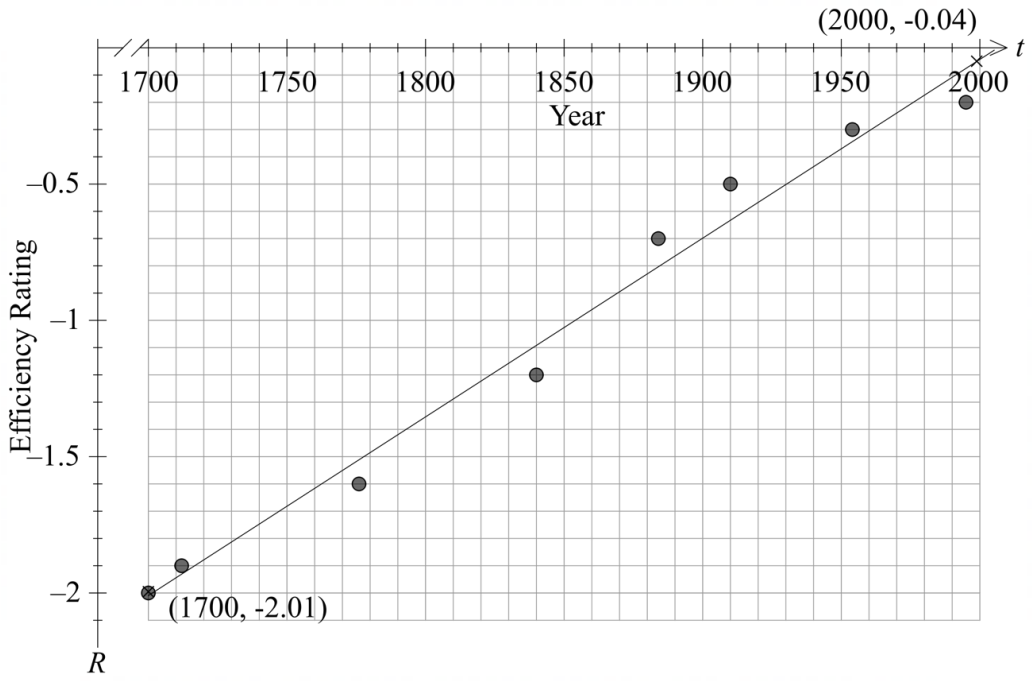
Question 10 **(12 marks)**

Combustion engines work by converting heat energy into kinetic energy. The efficiency rating, R, is based on the amount of heat energy converted into kinetic energy.

The table below gives the efficiency rating of several engines and the year they were invented. The most efficient an engine can be is when $R = 0$.

Engine	Savery	New-comen	Watt	Cornish	Parsons	Triple	Steam	Gas
Year (t)	1700	1712	1776	1840	1884	1910	1954	1995
Efficiency (R)	-2	-1.9	-1.6	-1.2	-0.7	-0.5	-0.3	-0.2

(a) Complete the scattergraph below by plotting the data for Steam and Gas engines. (1 mark)



Solution	**Specific behaviours**	**Point**
See above graph	✓ Plots last two points.	3.1.5

(b) Calculate the correlation coefficient r_{tR} for this data, correct to four decimal places. (1 mark)

Solution	**Specific behaviours**	**Point**
0.9909	✓ Calculates correlation coefficient.	3.1.7

(c) Determine the equation of the least-squares line for this data, with coefficients correct to four decimal places. (2 marks)

Solution	**Specific behaviours**	**Point**
$R = 0.0066t - 13.1546$	✓ States least-squares line. ✓ Uses correct variables and rounds to two decimal places.	3.1.10

See next page

(d) Plot the least-squares line on the above scattergraph, indicating any points used. (2 marks)

Solution	Specific behaviours	Point
$t = 1700,\ \hat{R} = -2.01$ $t = 2000,\ \hat{R} = -0.04$	✓ States two points use. ✓ Draws in line.	3.1.10

(e) (i) What part of the least-squares line in part (c) indicates that engines are becoming more efficient over time? (1 mark)

Solution	Specific behaviours	Point
The positive gradient	✓ States positive gradient.	3.1.12

(ii) Interpret your answer to part (e)(i) in the context of this question. (1 mark)

Solution	Specific behaviours	Point
Each year the efficiency increases by approximately 0.0066	✓ Interprets gradient correctly.	3.1.12

A new combustion engine is being developed by a motor company and will be released in 2023.

(f) (i) Determine the predicted efficiency rating for this motor. (1 mark)

Solution	Specific behaviours	Point
$\hat{R} = 0.1079$	✓ Predicts efficient rating.	3.1.14

(ii) Comment on the validity of this prediction, giving **two** justifications for your answer. (3 marks)

Solution	Specific behaviours	Point
Prediction is unreliable Extropolation has been used and $\hat{R} > 0$ which is not possible in this model	✓ States prediction is unreliable. ✓ States extrapolation has been used. ✓ Gives a second suitable justification.	3.1.15

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Question 11 **(16 marks)**

Mark uses mustard in his sandwiches. He places the same amount of mustard in each sandwich.

Mark opened a new jar of mustard. By the end of the fourth day there was 165 g of mustard left in the jar, and at the end of the seventh day there was 120 g of mustard left in the jar.

(a) Determine the amount of mustard that Mark uses in each sandwich. (2 marks)

Solution	Specific behaviours	Point
$\frac{165-120}{4-7}=-15$ i.e. 15 g is used in each sandwich	✓ Determines common difference. ✓ Interprets the common difference in the context of the question.	3.2.4

(b) Determine the amount of mustard in a new jar. (1 mark)

Solution	Specific behaviours	Point
$165+4\times 15=225$ g	✓ Determines amount of mustard.	3.2.3

(c) Write down a recursive rule for the amount of mustard remaining in the jar, M_n, at the end of nth day. (2 marks)

Solution	Specific behaviours	Point
$M_{n+1}=M_n-15 \quad M_0=225$	✓ Writes a suitable recursive rule using the common difference from (a). ✓ Includes a starting term (either M_0 or M_1).	3.2.1

(d) Determine the number of sandwiches that Mark can make from a jar of mustard. (2 marks)

Solution	Specific behaviours	Point
$M_{15}=0$ g i.e. 15 sandwiches	✓ Uses technology or another procedure. ✓ States number of sandwiches.	3.2.1 3.2.4

Mark buys 4 jars of mustard each quarter (every three months). He notices that the cost of his mustard at The Potato Barn increases by 2.3% every quarter. In the first quarter of 2023 he spent $\$5$ on each jar.

(e) (i) How much did Mark spend on mustard in the first quarter of 2023? (1 mark)

Solution	Specific behaviours	Point
$4\times \$5=\$20$	✓ Determines correct amount.	3.2.5

(ii) Hence, write down a recursive rule for the amount that Mark spends on mustard during a quarter, A_n, where A_1 is the first quarter of 2023. (1 mark)

Solution	Specific behaviours	Point
$A_{n+1}=1.023\,A_n \quad A_1=20$	✓ Writes a suitable recursive rule.	3.2.5

See next page

(f) Determine how much Mark spends on mustard during 2023. (2 marks)

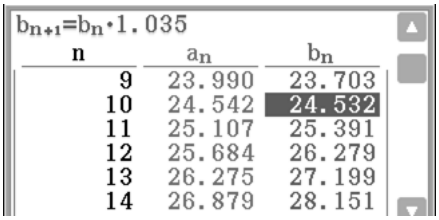
Solution	Specific behaviours	Point
$\text{Sum} = \$82.80$	✓ Determines sum of the first four terms. ✓ Rounds correctly to two decimal places.	3.2.8

(g) Determine during which quarter and year Mark will start paying \$7 a jar. (3 marks)

Solution	Specific behaviours	Point
$\$7 \times 4 = \$28$ for the quarter $A_{16} = \$28.13$ i. e. during the last quarter of 2026	✓ Determines cost for the quarter, or uses $A_{n+1} = 1.023\,A_n \quad A_1 = 5$. ✓ Determines cost exceeds \$7 after 16 quarters. ✓ States quarter and year.	3.2.8

Mark finds out that Aldente sells the same mustard for \$4.50 a jar in the first quarter of 2023. However, their prices are increasing by 3.5% each quarter.

(h) For how long will Aldente be cheaper than The Potato Barn. (2 marks)

Solution	Specific behaviours	Point
In Aldo: $A_{n+1} = 1.035A_n \quad A_1 = 18$  $b_{n+1}=b_n \cdot 1.035$ n | a_n | b_n 9 | 23.990 | 23.703 10 | 24.542 | 24.532 11 | 25.107 | 25.391 12 | 25.684 | 26.279 13 | 26.275 | 27.199 14 | 26.879 | 28.151 i. e. Aldo is cheaper up to the second quarter of 2025	✓ Determines a recursive rule for the price in Aldo. ✓ States quarter and year.	3.2.5 3.2.8

Question 12 **(10 marks)**

A primary school in a new suburb has capacity for 300 students. The Education Department predicts that each year, the number of new students is 70, and the number of students who leave during the year will be 6.5% of the number of students at the start of the year. Each year, 40 students leave at the end of the year. There are currently 210 students in the primary school.

Let S_n be the number of students in the school at the start of the year, n years from 2023.

The number of students can be represented by the equation

$$S_{n+1} = aS_n + b, \quad S_0 = 210$$

(a) Explain why $a = 0.935$. (1 mark)

Solution	Specific behaviours	Point
6.5% of students leave during the year i. e. 93.5% stay	✓ Explains in the context of the question.	3.2.10

(b) Determine the value of b. (1 mark)

Solution	Specific behaviours	Point
$b = 70 - 40 = 30$	✓ Determines b.	3.2.10

(c) Determine the number of students in the primary school at the start of each of the next three years. (2 marks)

Solution	Specific behaviours	Point
$S_1 = 226.35$ i. e. 226 $S_2 = 241.64$ i. e. 241 $S_3 = 255.93$ i. e. 255	✓ Determines first three terms. ✓ Rounds values consistently.	3.2.9

The Education Department plans to extend the school in two stages by building new facilities. The first stage will increase capacity by 80 students by the end of 2029.

(d) Is the completion of the first stage fast enough? Justify your answer. (2 marks)

Solution	Specific behaviours	Point
$S_7 = 304$ students > 300 School exceeds current capacity by start of 2030. Hence the first stage will be completed in time.	✓ Determines in which year students exceed 300. ✓ Concludes the first stage is completed in time.	3.2.9

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The second stage will be completed when the school reaches the new capacity of 380, and will increase capacity by a further x students.

(e) By when should the second stage be completed? Justify your answer. (2 marks)

Solution	Specific behaviours	Point
$S_{17} = 381$ students > 380 Second stage needs to be completed by end of 2039 $(Beginning\ of\ 2040)$	✓ Determines in which year students exceed 380. ✓ States year when second stage needs to be completed by.	3.2.11

The Education Department wishes for the second stage to be big enough to cater for any future growth.

(f) Determine the value of x, justifying your answer. (2 marks)

Solution	Specific behaviours	Point
From calculator stead state is approaching 462 Hence $x = 462 - 380$ $= 82$ students	✓ Uses technology to determine steady state solution. ✓ Determines value of x.	3.2.10 3.2.11
Alternative Solution	**Specific behaviours**	**Point**
$S = 0.935S + 30$ $S = 461.54$ i. e. 462 students Hence $x = 462 - 380$ $= 82$ students	✓ Determines steady state solution. ✓ Determines value of x.	3.2.10 3.2.11

See next page

Question 13 **(13 marks)**

(a) A survey of surgeons ranked their video game skills (x) with how they performed in simulated surgery (y). The data showed a strong positive correlation. The survey then concluded that playing more video games makes you a better surgeon.

(i) Explain whether the data supports this statement. (2 marks)

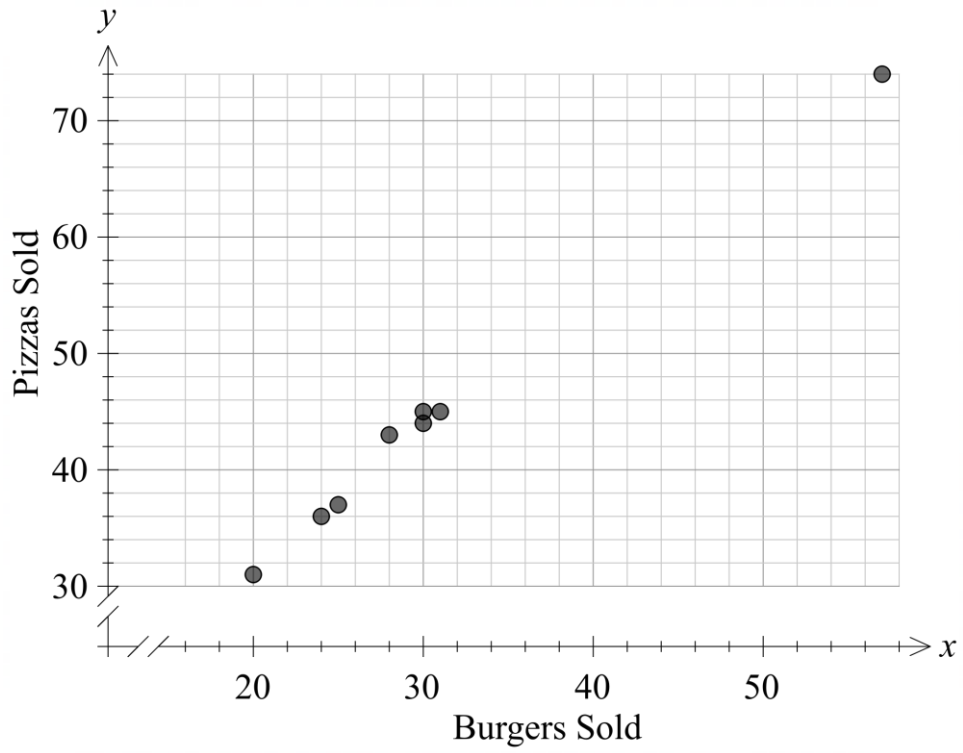
Solution	Specific behaviours	Point
Statement is no supported. While the data shows there is a correlation between the two variables, correlation does not imply causation.	✓ States statement is not supported. ✓ Mentions correlation does not imply causation.	3.1.7

(ii) Identify a possible explanation for the association noticed. (2 marks)

Solution	Specific behaviours	Point
There could be a confounding variable. For example, manual dexterity could explain both variables.	✓ States there is a third variable or confounding variable. ✓ Gives an example of a suitable third variable.	3.1.8

(b) A fast food store owner recorded the daily number of burgers and pizzas sold at their restaurant. The following table and scattergraph shows this data for the last eight days.

Burgers (x)	20	24	25	28	30	30	31	57
Pizzas (y)	31	36	37	43	45	44	45	74



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The correlation coefficient of this data is $r_{xy} = 0.9967$.

(i) Comment on the association between the two variables in terms of direction and strength. (2 marks)

Solution	Specific behaviours	Point
There is a strong positive linear correlation between the amount of burgers and pizzas sold.	✓ States the strength is strong. ✓ States the direction is negative.	3.1.6

The point $(57,74)$ was on a day when there was a festival. The restaurant owner feels that this is an outlier and removes the point. The correlation coefficient is recalculated as $r_{xy} = 0.991$.

(ii) Explain why the correlation coefficient has not changed significantly despite the removal of an outlier. (2 marks)

Solution	Specific behaviours	Point
The point $(57,74)$ was following the trend in the data. The remaining data still has a strong positive association.	✓ Indicates the point was following the trend in the data. ✓ States remaining data still has a strong positive association.	3.1.6

(iii) Which of the following (**A**, **B** or **C**) would be the least squares regression line for this data.

A $\hat{y} = 3.78 - 1.35x$ **B** $\hat{y} = 3.78 + 1.35x$ **C** $\hat{y} = 3.78 + 13.5x$

Justify your choice by referring to two features shown on the scatter graph. (3 marks)

Solution	Specific behaviours	Point
The least squares regression line is **B**. The correlation coefficient is positive, and hence gradient of the line is positive. The gradient in **C** is too steep.	✓ States the line is **B**. ✓ Uses the correlation coefficient to justify that the gradient is positive. ✓ States gradient of **C** is too steep.	3.1.10

The restaurant owner decided to advertise and promote one of their products. The data for another week of sales produced the following:

- a least squares regression line of $\hat{y} = 5.76 + 2.71x$
- correlation coefficient of 0.9945

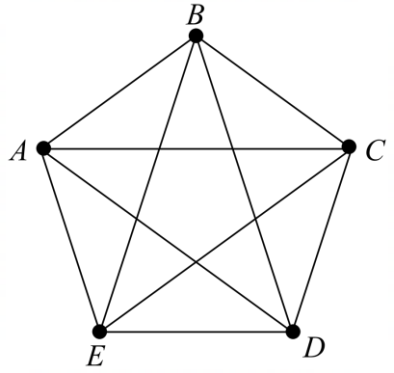
(iv) Determine which product they promoted. Justify your answer by referring to the data above. (2 marks)

Solution	Specific behaviours	Point
They promoted Pizzas. The gradient is much steeper, so for every burger sold they are now selling more pizzas.	✓ States pizzas were promoted. ✓ Explains using the gradient of the least squares regression line.	3.1.12

See next page

Question 14 **(11 marks)**

The complete graph with five vertices, K_5, is shown below.



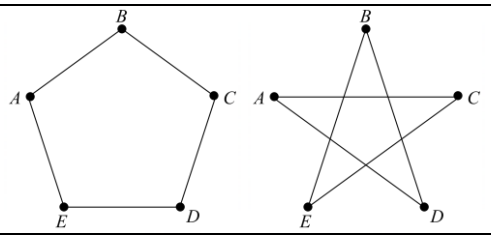
(a) Explain why K_5 is Hamiltonian. (1 mark)

Solution	Specific behaviours	Point
There is a closed path that visits every vertex and returns to the start.	✓ Explains why K_5 is Hamiltonian.	3.3.9

(b) (i) Identify a Hamiltonian cycle in K_5. (1 mark)

Solution	Specific behaviours	Point
$ABCDEA$ or $ACEBDA$	✓ States a Hamiltonian cycle.	3.3.9

(ii) Remove this Hamiltonian cycle from K_5 and draw the resulting subgraph. (1 mark)

Solution	Specific behaviours	Point
	✓ Draws one of the two graphs shown.	3.3.1

(iii) Is the subgraph in part (b)(ii) Hamiltonian? Justify your answer. (2 marks)

Solution	Specific behaviours	Point
Yes, the Hamiltonian cycle is $ABCDEA$ or $ACEBDA$	✓ States yes. ✓ States a Hamiltonian cycle.	3.3.9

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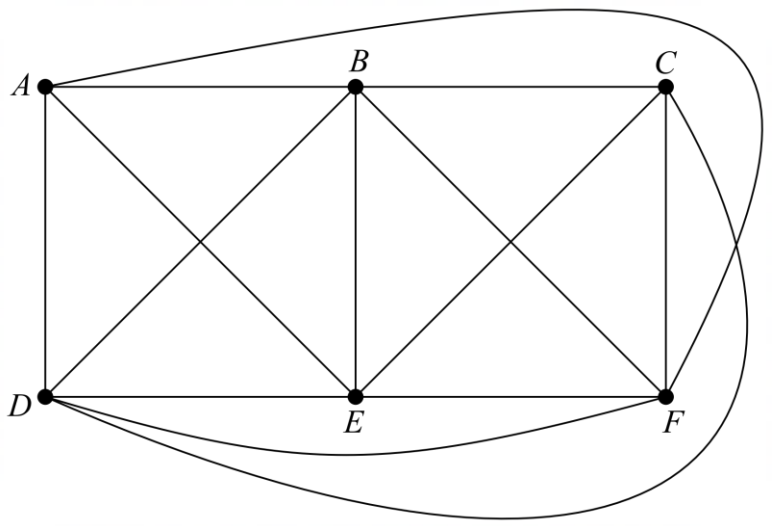
The complete graph with four vertices, K_4, is also Hamiltonian.

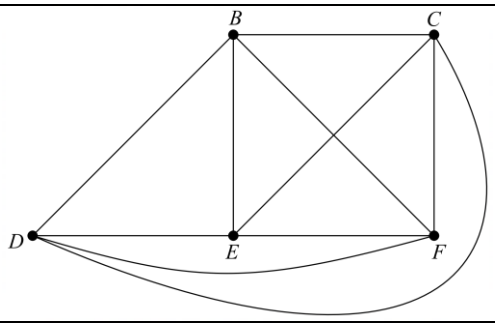
(c) If a Hamiltonian cycle is removed from K_4 is the resulting subgraph Hamiltonian? Explain your answer. (3 marks)

Solution	Specific behaviours	Point
No, as the resulting subgraph is disconnected, and hence is not Hamiltonian.	✓ States no. ✓ States subgraph is disconnected. ✓ States it is not Hamiltonian.	3.3.1

Polish mathematician, Kasimir Kuratowski, proved that if K_5 was a subgraph of a larger graph, then the larger graph cannot be planar.

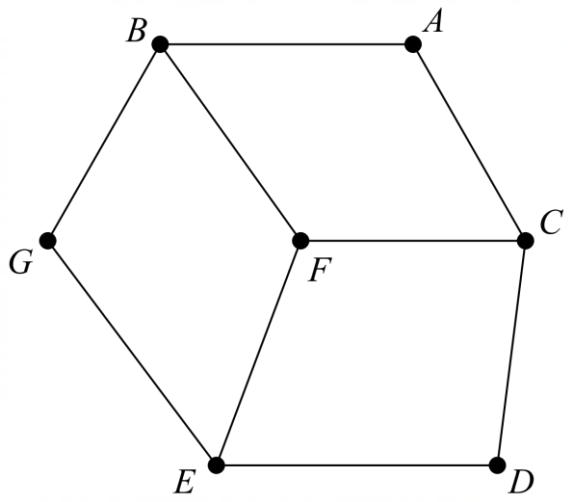
(d) Show that the graph below is not planar, by drawing a suitable subgraph below. (3 marks)



Solution	Specific behaviours	Point
	✓ Graph drawn is a subgraph of the original. ✓ Graph has five vertices. ✓ Graph is K_5.	3.3.1

Question 15 **(9 marks)**

The diagram shows the air conditioning ducting installed in the roof of a warehouse.



A tradesperson is employed to inspect the ducting.

They must enter the roof through a manhole at B.

The tradesperson needs to inspect each section of ducting at least once and wishes to minimise the number of sections inspected more than once.

(a) (i) Explain why the tradesperson will need to inspect some sections more than once. (2 marks)

Solution	Specific behaviours	Point
As the graph is not Eulerian, they will need to inspect some sections more than once to return to B.	✓ States the graph is not Eulerian. ✓ Explains the need to repeat sections to return to B.	3.3.8

(ii) Suggest a route by which the tradesperson can inspect each section of ducting, minimising the number of sections inspected more than once. State the sections which are inspected more than once. (3 marks)

Solution	Specific behaviours	Point
$BACDEFCABGEFB$ Repeated edges are BA, AC, EF	✓ States a possible route. ✓ Length of route is 12. ✓ States edges that are repeated.	3.3.6

(b) A new piece of ducting is added between C and E. What effect will this have on the route you found in part (a)(ii)? Justify your answer. (3 marks)

Solution	Specific behaviours	Point
New edge means the graph is now Semi − Eulerian. They will only need to inspect BF twice as this is the only edge joining the two odd vertices. New route: $BGEDCEFCABFB$	✓ States graph is now Semi-Eulerian. ✓ Indicates BF needs is only edge to be repeated. ✓ States a new route (length = 11).	3.3.8

(c) What feature of the graph indicates that vertex F must be visited twice? (1 mark)

Solution	Specific behaviours	Point
$\text{degree}(F) = 3$	✓ States the degree.	3.3.1

End of questions

Supplementary page

Question number: _____